Semi-open and semi-closed set in Bitopological Spaces

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Abstract
In this paper we define semi-open set and semi-closed set in bitoplogical space due to [Ratnesh Kumar Saraf ,2000] which is used to define generalized continuous function , semi-generalized continuous function , generalized semi- continuous function and ψ-continuous function in bitopological spaces . some of the fundamental properties and relationship between these continuous functions are investigated in this paper.

1-introduction
Let (X,p_1,p_2)be a bitopological space ,a subset D of X is say to be open set (resp. closed set) in the bitopological spae X if it is open(resp. closed)set in (X,p_1) or (X,p_2). the concept of semi-open set in topological spaces was introduced in 1963by [N.Levine,1963]. [Levine, 1970]generalized the concept of closed sets to generalized closed sets . [Bhattacharya and Lahiri,1987] generalized the concept of closed sets to semi-generalized closed sets via semi-open sets .the complement of a semi-open(resp. semi-generalized-closed)set is called semi closed set [N.Levine,1970] (resp.semi-generalized open).

[Kumar , 1991] introduced and defined a maps namely ψ-continuity and he discus the relation between this map and semi –continuity [Biswas,N ,1970], generalized-continuity[Caldas,M ,1993], [R.Devi, H.Maki and K.Balachandran ,1993], [K.Balachandran ,P.Sundaram and H.Maki,1991], semi generalized -continuity[P.sundaram, H.Maki, K. Balachandran,1991], [P.Bhattacharya and B.K.Lahiri 1987] and generalized semi-continuity [Miguel Caladas Cueva and Ratnesh Kumar saraf ,2000].

The purpose of this paper that is give a new definition of these concepts in bitopological space$(X,p_1,p_2)$with some of its theorems and properties.

2- basic definition
Definition(2-1)
Let (X,p_1,p_2) be a bitopological space then a subset A of X is say to be

a- semi-open set if $A\subseteq cl_{pi}(int_{pi}(A))$ and semi-closed if $int_{pi}(cl_{pi}(A)) \subseteq A$ for i=1or2.

b- generalized-closed set(g-closed) iff $cl_{pi}(A) \subseteq U$ where $A\subseteq U$ and U is pi-open set in (X,p_i) for i=1or2.

c- semi-generalized closed (sg-closed set) if $scl_{pi}(A) \subseteq U$ whenever $A\subseteq U$ and U is semi-open set in (X,p_i) for i=1or2

d- generalized semi-closed(gs-closed) iff $scl_{pi}(A) \subseteq U$,where $A\subseteq U$ and U is open set in (X,p_i) for i=1or2.

e- ψ-closed set if $scl_{pi}(A) \subseteq U$ whenever $A\subseteq U$ and U is sg-open set in (X,p_i) for i=1or2

Remark(2-2)

1-The class of ψ-closed sets properly contains the class of closed sets .
2-The class of ψ-closed sets is properly contained in the class of sg-closed sets and contained in the class of gs-closed sets .

Theorem(2-3)

Every semi closed set is ψ-closed set and the converse is not true
Proof: exist by definition

Example(2-4)

Let $X=\{a,b,c\}$ and $p_1=\{X, \emptyset,\{a,b\}\}$, $p_2=\{X, \emptyset,\{c\}\}$ then $A=\{a,c\}$ is ψ-closed but not semi closed

Theorem(2-5)

a subset A of X is semi-open set (respectively, semi-closed , sg-closed, gs-closed ,g-closed,ψ-closed) set in (X,p_i) for i=1or2 then it is semi-open set(respectively ,semi-closed ,sg- closed , gs-closed ,g-closed , ψ-closed) set in (X,p_1,p_2)
proof:
exist by definition

Remark(2-6):[M.K.R.S.Veera Kumar,1991]

1-every semi closed set and thus every pi-closed set is ψ-closed set
2- every ψ-closed is sg-closed set and also gs-closed set
3- every semi-closed set is sg-closed set

Example(2-7)

let $X=\{a,b,c\}$, $p_1=\{X,\emptyset,\{a\},\{b,c\}\}$, $p_2=\{X, \emptyset,\{a,b\}\}$
$\{a,c\}$ is sg-closed but not semi closed set .

Definition (2-8)[Miguel Caldas Cueva1995**]**

For any subset E of (X,p_1,p_2) $scl_{pi}*(E)= \cap\{A:E\subseteq A$ such that $A\in sd(X,p_1,p_2)\}$ where $sd(X,p_1,p_2)=\{A:A\subset X$ and A is sg-closed in (X,p_1,p_2) $\}$ and $SO(X,p_1,p_2)^* =\{B:scl_{pi}*(B^c)=B^c\}$ for i=1or2.

Theorem(2-9)

A bitopological space (X,p_1,p_2) is semi $T_{1/2}$ -space if and only if for each $x\in X$, $\{x\}$ is semi open or semi closed
Proof
Suppose that for some $x \in X$, $\{x\}$ is not semi closed .since X is the only semi open set containing $\{x\}^c$, the set $\{x\}^c$ is sg-closed set so it is semi closed set in the semi$T_{1/2}$ space (X,p_1,p_2) ,there for $\{x\}$ is semi open set
Conversely , since $SO(X,p_1,p_2) \subseteq SO(X,p_1,p_2)^*$ holds it is enough to prove that $SO(X,p_1,p_2)^*\subseteq SO(X,p_1,p_2)$. let $E \in SO(X,p_1,p_2)^*$ .suppose that $E \notin SO(X,p_1,p_2)$.then $scl_{pi}*(E^c)=E^c$ and $scl_{pi}(E^c)\neq E^c$ hold. There exist a point x of x such that $x\in scl_{pi}(E^c)$ and $x \notin E^c(=scl_{pi}*(E^c))$.since $x\notin sc_{pi}*(E^c)$ there exist sg-clsed set A such that $x\notin A$ and $E^c\subset A$ .by the hypothesis the singleton $\{x\}$ is semi-open set or semi-closed set.

Now if $\{x\}$ is semi-open set ,since $\{x\}^c$ is semi closed set with $E^c \subset \{x\}^c$, we have $scl_{pi}(E^c) \subset \{x\}^c$, i.e, $x \notin scl_{pi}(E^c)$. this contradicts the fact that $x \in scl_{pi}(E^c)$. there for $E \in SO(X,p_1,p_2)$.
If $\{x\}$ is semi closed set , since $\{x\}^c$ is semi-open set containing the sg-closed set A ($\supset E^c$) we have $scl_{pi}(E^c) \subset scl_{pi}(A) \subset \{x\}^c$. there for $x \notin sclpi(E^c)$. this is contradiction . there for $E \in SO(X,p_1,p_2)$.
Hence in both case we have $E \in SO(X,p_1,p_2)$, i.e, $SO(X,p_1,p_2)^* \subseteq SO(X,p_1,p_2)$.

3-the continuity

Definition(3-1)

A function f from a bitopological space (X,p_1,p_2) into a bitopological space (Y,w_1,w_2) is called continuous if and only if $f^{-1}(V)$ is pi-open set (pi-closed set) in X for each wi-open set (wi-closed set)in Y and it is called semi continuous if $f^{-1}(V)$ is semi open(semi-closed) set for each pi-open(pi-closed) set V in Y

Example(3-2)

Let $X=\{a,b,c\}=Y$, $p_1=\{X,\varnothing,\{b\}\}$,$p_2=\{X,\varnothing,,\{a\}\}$
$W_1=\{Y, \varnothing,\{a\},\{a,c\}\}$, $w_2=\{Y, \varnothing,\{a\}\}$ then
$f:(X,p_1,p_2)\to(Y,w_1,w_2)$defined by f(a)=a is semi-continuous mapping.

Remark (3-3)

If $f:(X,p_i) \to(Y,w_i)$ is semi- continues for i=1or2 then f: $(X,p_1,p_2) \to(Y,w_1,w_2)$ is not necessary semi- continues

Example(3-4)

Let $X=\{a,b,c\}$, $p_1=\{X,\varnothing,\{a,c\}\}$,$p_2=\{X, \varnothing,\{a\}\}$ and $Y=\{e,d,g\}$, $w_1=\{Y, \varnothing, \{e\}\}$, $w_2=\{Y, \varnothing, \{d,g\}\}$, let $f:(X,p_i)\to(Y,w_i)$ defined by f(a)=d , f(b)=f(c)=g then f is semi-continuous but $f:(X,p_1,p_2)\to(Y,w_1,w_2)$ is not semi-continuous since $f^{-1}(\{e,g\})=\{b,c\}$ which is not semi-open set in (X,p_1,p_2)

Remark(3-5)

The composition of two semi-continuous functions is not necessary semi continuous.

Example(3-6)

Let $X=\{a,b,c\}$, $p_1=\{X, \varnothing, \{ a\},\{a,c\}\}$, $p_2=\{ X, \varnothing,\{a,b\}\}$ and $Y=\{1,2,3\}$, $w=\{Y, \varnothing,\{1\},\{2\},\{1,2\}\}$, $w_2=\{Y, \varnothing,\{2\}\}$, $Z=\{e,d,g\}$, $k_1=\{Z, \varnothing,\{e\}\}$,$k_2=\{Z, \varnothing,\{d,g\}\}$ let f $:(X,p_1,p_2)\to(Y,w_1,w_2)$ and $h:(Y,w_1,w_2)\to(Z,k_1,k_2)$ definef by f(a)=1,f(b)=3,f(c)=2, h(1)=e , h(2)=g , h(3)=d then f , h is semi-continuous but hof is not semi-continuous since $(hof)^{-1}(\{e,g\})=\{a,c\}$ which is not semi-open set in (X,p,p).

Theorem(3-7)

If f: $(X,p_1,p_2) \to(Y,w_1,w_2)$ is semi continues and g: ($Y,w_1,w_2) \to(Z,s_1,s_2)$ is continues then gof is semi continues
Proof
Let A is si-open set in Z . since g is continues then $g^{-1}(A)$ is wi-open set in Y . since f is semi-continues then $f^{-1}(g^{-1}(A))$ is semi-open set in X and then gof is semi-continues.

Definition(3-8)

A function f from a bitopological space (X,p_1,p_2) into a bitopological (Y,w_1,w_2) is called generalized-continues if $f^{-1}(V)$ is g-closed in X for every wi-closed set in Y.

Example(3-9)

Let $X=\{a,b,c\}=Y$, $p_1=\{X,\varnothing,\{b\}\}$, $p_2=\{X,\varnothing,\{a\}\}=w_2$, $w_1=\{Y,\varnothing,\{a\},\{a,c\}\}$ then $f:(X,p_1,p_2)\rightarrow(Y,w_1,w_2)$ is g-continuous

Theorem(3-10)

$f: (X,p_i) \rightarrow(Y,w_i)$ is g-continues for i=1and2 if and only if $f: (X,p_1,p_2) \rightarrow(Y,w_1,w_2)$ is g-continues

Proof:

The proof exist from the fact that every closed set and g-closed in bitopology exist in (X,p_i) and (Y,w_i) respectively for i=1or 2

Remark (3-11)

The composition of two generalized -continuous functions is not necessary g-continues.

Example(3-12)

Let $X=\{a,b,c\}=Y$, $p_1=\{X,\varnothing,\{b,c\},\{a\}\}$, $p_2=\{X,\varnothing,\{a,b\}\}$,$w_2=\{Y,\varnothing,\{a\}\}$, $w_1=\{Y,\varnothing,\{a,c\}\}$ and $Z=\{e,d,g\}$, $k_1=\{Z, \varnothing,\{e\}\}$, $k_2=\{Z, \varnothing,\{d,g\}\}$let $f:(X,p_1,p_2)\rightarrow(Y,w_1,w_2)$ definef by f(a)=a , f(b)=c , f(c)=b and h: $(Y,w_1,w_2)\rightarrow(Z,k_1,k_2)$ defined by h(a)=h(b)=e , h(c)=d then f and h are g-continuous but hof is not g-continuous since $(hof)^{-1}(\{e\}=\{a,c\}$ which is not g-closed set in (X,p_1,p_2)

Definition(3-13)

A function $f:(X,p_1,p_2)\rightarrow(Y,w_1,w_2)$ is called semi-generalize continues function if $f^{-1}(V)$ is sg-closed in X for each g-closed set in Y

Theorem(3-14)

$f:(X,p_i) \rightarrow(Y,w_i)$ is sg- continuous if and only if $f: (X,p_1,p_2) \rightarrow(Y,w_1,w_2)$ is sg- continuous

Proof:

Let F is g-closed set in (Y,w_1,w_2) then F is g-closed in (Y,wi),i=1or2, since f is sg-continues $f^{-1}(F)$ is sg-closed in (X,p_i) and by theorem(3-5) $f: (X,p_1,p_2) \rightarrow(Y,w_1,w_2)$ f is sg- continues ,conversely, let B is g-closed set in (Y,wi) i=1or2 , by theorem (3-5) b is g-closed set in (Y,w_1,w_2) since f is sg- continuous $f^{-1}(B)$ is sg-closed set in $(X,p_1,p_2)$and by the same theorem $f^{-1}(B)$ is pi-sg-closed set then $f:(X,p_i) \rightarrow(Y,w_i)$ is sg- continuous.

Remark(3-15)

The composition of two sg- continuous in bitopological space is not necessary sg- continues

Example (3-16)

Let $X=\{a,b,c\}$, $p_1=\{X,\varnothing,\{a,c\}\}$, $p_2=\{X,\varnothing,\{a\}\}$, and let $Y=\{1,2,3\}$, $w_1=\{Y,\varnothing,\{1\},\{2\},\{1,2\}=w_2$ and $Z=\{e,d,g\}$, $k_1=\{Z, \varnothing,\{e\}\}$, $k_2=\{Z, \varnothing,\{d,g\}\}$ let $f:(X,p_1,p_2)\rightarrow(Y,w_1,w_2)$ defined by f(a)=1 , f(b)=3 , f(c)=2 and h: $(Y,w_1,w_2)\rightarrow(Z,k_1,k_2)$ defined by h(3)=h(2)=d , h(1)=g then f and h are sg- continuous but hof is not sg-continues since $(hof)^{-1}(\{g\}=\{a\}$ which is not sg-closed set in (X,p_1,p_2)

Definition(3-17)

A function f from a bitopology space X into Y is called gs- continuous if $f^{-1}(V)$ is gs-closed for each semi-closed set V in Y

Theorem(3-18)

If $f: (X,p_i) \rightarrow(Y,w_i)$ is gs- continues then $f: (X,p_1,p_2) \rightarrow(Y,w_1,w_2)$ is gs- continues

Proof:

Let V is semi closed set in (Y,w_1,w_2) then it is semi closed in (Y,w_i) i=1or2 , since f is gs- continues then $f^{-1}(V)$ is gs-closed in (X,p_i) i=1or2 and it is gs-closed in (X,p_1,p_2), then f is gs – continuous

Remark(3-19)

The composition of two gs- continuous function is not necessary gs- continuous

Example(3-20)

Let X={a,b,c} , p_1={X,∅,{a,c},{a}} , p_2={X,∅,{a,b}}, and let Y={1,2,3} , w_1={Y,∅,{1},{2},{1,2},w_2={Y, ∅,{2}} and Z={e,d,g} , k_1={Z, ∅,{e}}, k_2={Z, ∅,{d,g}} let $f:(X,p_1,p_2)\rightarrow(Y,w_1,w_2)$ define f by f(a)=1 , f(b)=3 , f(c)=2 and h: $(Y,w_1,w_2)\rightarrow(Z,k_1,k_2)$ defined by h(1)=d,h(2)=e , h(3)=g then f and h are gs- continuous but hof is not gs- continuous since $(hof)^{-1}$({f,g}={a,b} which is not gs-closed set in (X,p_1,p_2)

Definition(3-21)

A function f from a bitopological space X into a bitoplogical space y is called ψ-continues if $f^{-1}(V)$ is ψ-closed set for every pi-closed set V in Y

Remark(3-22)

If $f:(X,p_i)\rightarrow(Y,w_i)$ is ψ-continues then it is not necessary that
$f: (X,p_1,p_2)\rightarrow(Y,w1,w2)$ is ψ-continues

Example(3-23)

Let X={a,b,c} , p_1={X,∅,{a},{c,b}}= p_2={X, ∅,{{a,b} and let Y={1,2,3} , w_1={Y,∅,{1},{2},{1,2},w_2={Y, ∅,{2}} , now f: (x,pi)→(y,wi) defined by f(a)=1= f(b) , f(c)=3 is ψ-continuous but $f:(X,p_1,p_2)\rightarrow(Y,w_1,w_2)$ is not ψ-continuous since f^{-1}({1}={a,b} which is not ψ-closed set in (X,p_1,p_2)

Theorem(3-24)

If $f:(X,p_1,p_2)\rightarrow(Y,w_1,w_2)$ is continues then f is ψ-continuous

Proof

Let V is wi-closed set in y then $f^{-1}(V)$ is pi-closed set in X and since every pi-closed set is ψ-closed set then f is ψ-continuous.

The converse of the above theorem is not true as we show in the following example

Example(3-25)

Let X={a,b,c} , p_1={X,∅,{a},{b},{a,b}} , p_2={X,∅,{a}}
W_1={Y,∅,{a},{a,c}} , w_2={Y,∅,{b}}
Then f: $(X,p_1,p_2)\rightarrow(Y,w_1,w_2)$ defined by f(x)=x is ψ-continuous but not continues.

Remark(3-26)

The composition of two ψ-continuous function is not necessary w-continuous

Example(3-27)

Let X={a,b,c} , p_1={X,∅,{a,b}}= p_2 and let Y={1,2,3} , w_1={Y,∅,{1},{2},{1,2},w_2={Y, ∅,{2}} and Z={e,d,g} , k_1={Z, ∅,{e}}, k_2={Z, ∅,{d,g}} let $f:(X,p_1,p_2)\rightarrow(Y,w_1,w_2)$ definef by f(a)=1 , f(b)=3 , f(c)=2 and h: $(Y,w_1,w_2)\rightarrow(Z,k_1,k_2)$ defined by h(1)=h(3)=d , h(2)=g then f and h are ψ-continuous but hof is not ψ-continuous since $(hof)^{-1}$({f}={b} which is not ψ-closed set in (X,p_1,p_2)

The following diagram shows the relationships established between the above types of continuity

continuous→ semi- continuous→ψ-continuous→ gs - continuous

Theorem(3-28)

If $f:(X,p_1,p_2)\to(Y,w_1,w_2)$ isψ-continuous and sg- continuous such that X is semi-$T_{1/2}$ space then f is continuous

Proof: exist by definition

Example(3-29)

Let $X=\{a,b,c\}$, $p_1=\{X, \emptyset, \{b\}\}$, $p_2=\{X,\emptyset,\{a\}\}=w_2$, $w_1=\{Y, \emptyset,\{a\},\{a,c\}$ and let $f:(X,p_1,p_2)\to(Y,w_1,w_2)$ defined by f(a)=b , f(b)=c , f(c)=a then f is gs- continuous but not ψ- continuous since $f^{-1}(b)=\{a\}$ which is not ψ-closed in(Y,w_1,w_2) .

Theorem(3-30)

If $f:(X,p_1,p_2)\to(Y,w_1,w_2)$ is ψ-continues such that Y is $T_{1/2}$ space then f is sg-continuous

Proof:

Let A is g-closed set in Y . since Y is $T_{1/2}$ space A is ∂-closed and since f is ψ-continues $f^{-1}(A)$ is ψ-closed set in X and by remark (3-9) $f^{-1}(A)$ is sg-closed set and then f is sg- continues.

Theorem(3-31)

If $f:(X,p_1,p_2)\to(Y,w_1,w_2)$ is gs- continues such that X is semi$T_{1/2}$ space then f is semi-continues

Proof:

Let A is wi-closed set in Y . since f is gs- continues $f^{-1}(A)$ is gs-closed set in X . by remark $f^{-1}(A)$ is semi-closed set in X and then f is semi-continues

Theorem(3-32)

If $f:(X,p_1,p_2)\to(Y,w_1,w_2)$ is gs- continues such that Y is $T_{1/2}$ space then f is sg-continuous.

Proof:

Let A is g-closed set in Y .since Y is $T_{1/2}$ space A is ∂-closed set in Y and since f is gs- continues then $f^{-1}(A)$ is gs-closed set in X by remark(every gs-closed set is sg-closed set) we get that $f^{-1}(A)$ is sg-closed set , then f is sg- continuous

Theorem(3-32)

If $f:(X,p_1,p_2)\to(Y,w_1,w_2)$ is gs- continues such that X is semi$T_{1/2}$ space then f is ψ-continues

Proof:

Let A is wi-closed set in Y .since f is gs- continuous $f^{-1}(A)$ is gs-closed set , since X is semi$T_{1/2}$ space $f^{-1}(A)$ is semi-closed set and then by remark(3-9) $f^{-1}(A)$ is ψ-closed set in X .then f is ψ-continuous.

Theorem(3-33)

If $f:(X,p_1,p_2)\to(Y,w_1,w_2)$ is continuous such that X,Y are $T_{1/2}$ -space then f is sg-continuous

Proof:

Let A is g-closed set in Y .since Y is $T_{1/2}$ space then A is wi-closed set in Y and by hypothesis $f^{-1}(A)$ is pi-closed set in X and then it is sg-closed set in X then f is sg-continues

Example(3-34):

Let $X=\{1,2,3\}$, $p1=\{X, \emptyset,\{1\},\{2\},\{1,2\}\}$,$p2=\{X, \emptyset,\{2\}\}$

$Y=\{e,d,g\}$, $w1=\{Y, \emptyset,\{e\}\}$,$w2=\{Y, \emptyset,\{d,g\}\}$

Let $f:(X,p_1,p_2)\to(Y,w_1,w_2)$defined by f(1)=e , f(2)=g, f(3)=d then f is gs- continuous but not continuous since $f^{-1}(\{d,g\})=\{2,3\}$ which is not pi-closed set.

Example(3-35)

Let X={a,b,c} , p_1={X,∅,{a},{b},{a,b}} , p_2={X, ∅,{a}}
W_1={Y, ∅,{a},{a,c}} , w_2={Y, ∅,{b}}
Then f: $(X,p_1,p_2)\rightarrow(Y,w_1,w_2)$ defined by f(x)=x is semi-continues but not continuous.

Theorem(3-36)

If f:$(X,p_1,p_2)\rightarrow(Y,w_1,w_2)$ is sg- continues such that X is semi$T_{1/2}$ -space then f is ψ-continuous

Proof:

Let A is wi-closed set in Y by hypothesis $f^{-1}(A)$ is sg-closed set and then it is semi-closed set in X and by remark(3-9) it is ψ-closed set in X then f is ψ-continuous

Theorem(3-37)

If f:$(X,p_1,p_2)\rightarrow(Y,w_1,w_2)$ is g-continuous such that X is $T_{1/2}$ space then f is continuous

Proof

Let A is wi-closed set in Y then it is semi-closed set .since f is g-continuous $f^{-1}(A)$ is g-closed set in X and since X is $T_{1/2}$ space f is continuous.

Theorem(3-38)

If f:$(X,p_1,p_2)\rightarrow(Y,w_1,w_2)$ is g-continuous such that X is $T_{1/2}$ -space then f is gs- continuous

Proof

Let A is wi-closed set in Y . $f^{-1}(A)$ is g-closed set in X since X is $T_{1/2}$ then $f^{-1}(A)$ is pi-closed set in X and then it is gs-close set in X there for f is gs- continuous.

Theorem(3-39)

If f:$(X,p_1,p_2)\rightarrow(Y,w_1,w_2)$ is g-continuous such that X and Y are $T_{1/2}$ spaces then f is sg- continuous

Proof:

Let A is g-closed set in Y .since Y is $T_{1/2}$ space A is wi-closed set and then $f^{-1}(A)$ is g-closed set and since X is $T_{1/2}$ space $f^{-1}(A)$ is sg-closed set then f is sg- continuous.

Theorem(3-40)

If f:$(X,p_1,p_2)\rightarrow(Y,w_1,w_2)$ is g-continues such that X is $T_{1/2}$ space then f is ψ-continues

Proof:

Let A is wi-closed set in Y then A is semi-closed set and then $f^{-1}(A)$ is g-closed set in X .since X is $T_{1/2}$ space $f^{-1}(A)$ is pi-closed set and by remark (3-9) $f^{-1}(A)$ is ψ-closed set from that we get f is ψ-continuous

Theorem(3-41):

Let f:$(X,p_1,p_2)\rightarrow(Y,w_1,w_2)$ and h:$(Y,w_1,w_2)\rightarrow(Z,k_1,k_2)$ are two function hen:

1- hof is sg- continuous if f is sg-continuous and h is continuous function.
2- hof is gs-continuous if f is gs-continuous and h is continuous
3- hof is ψ-continues if f is ψ-continues and h is continuous.
4- hof is g-continuous if f is g-continuous and h is ψ-continues.

Proof: omitted .

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المجموعة شبه المفتوحة وشبه المغلقة في الفضاءات ثنائية التبولوجي

ملخص البحث

يتناول هذا البحث تعريف للمجموعة شبه المفتوحة وشبه المغلقة في الفضاء ثنائي التبولوجي (X,P_1,P_2).باستخدام هذا التعريف عرفنا بعض أنواع الدوال المستمرة التي تعتمد في تعريفها على المجموعة شبه المفتوحة او المجموعة شبه المغلقة وقمنا بدراسة بعض العلاقات بينها .